Multi-domain topology optimization of pulsed magnetic field generator sourced by harmonic current excitation

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Abstract — This paper presents a multi-domain topology optimization using the harmonically excited coil and the iron in order to focus pulsed magnetic field (PMF). The design sensitivity of the harmonic magnetic field is derived by adjoint variable method. As a result of the optimization, PMF is considerably concentrated on the objective domain with much less leakage than the initial model.

I. Introduction

Pulsed magnetic field (PMF) generated by harmonic current excitation is used for many applications such as the magnetic induction tomography [1], the pulsed electromagnetic field stimulator [2], the hyperthermia using induced eddy current [3], etc. However, most devices do not efficiently focus the PMF on the objective area so that much energy is undesirably wasted.

The topology optimization in the harmonic magnetic field has been researched [4], but the design of the multi-domain considering the coil carrying harmonic source current and the iron has never been presented.

In this paper, the method of the multi-domain topology optimization for concentrating PMF on the target position is introduced.

II. Multi-domain Topology Optimization of Harmonic Magnetic Field

A. Governing equation of harmonic magnetic field

A single governing equation for time harmonic magnetic field can be derived by a set of Maxwell's equations with respect to a complex vector potential , A^*, as follows:

$$\nabla \times (\frac{1}{\mu} \nabla \times A^*) + j\omega\sigma A^* = J_s, \tag{1}$$

where μ , ω , σ , J_s , and μ_0 are the permeability of material, the angular frequency, the electrical conductivity, the applied source current density, and the permeability in free space, respectively. In this paper, the motion of a conductor and permanent magnet are not considered for the analysis.

The variational equation is achieved by multiplying virtual vector potential $\bar{A}^*$ to the both sides of Eq. (1), and integrating over the 2D domain Ω as follows:

$$\begin{aligned} &\iint_\Omega \left[\nabla \times (\frac{1}{\mu} \nabla \times A^*) + j\omega\sigma A^* \right] \bar{A}^* d\Omega \\ &= \iint_\Omega J_s \cdot \bar{A}^* d\Omega, \text{ for all } \bar{A}^* \in \tilde{A}^*, \end{aligned} \tag{2}$$

where $\tilde{A}^*$ is the admissible vector potential.

Eq. (3) is obtained after applying Dirichlet and Neumann boundary conditions [5].

$$\begin{aligned} &\iint_\Omega \left[(\nabla \times A^*) \cdot (\frac{1}{\mu} \nabla \times \bar{A}^*) + j\omega\sigma A^* \bar{A}^* \right] d\Omega \\ &= \iint_\Omega J_s \cdot \bar{A}^* d\Omega, \end{aligned} \tag{3}$$

where the left side of Eq. (3) is an energy bilinear form $a\left(A^*, \bar{A}^*\right)$, and the right side is a load linear form $l\left(\bar{A}^*\right)$.

The matrix form of harmonic magnetic field is expressed as [5]

$$\left([K] + j\omega[M]\right)\left\{A^*\right\} = \{F\}. \tag{4}$$

For the 4-node finite element method, each term of an element in Eq. (4) is as follows:

$$\begin{aligned} [K] &= \frac{1}{\mu} \iint_\Omega \frac{\partial N^T}{\partial x}\frac{\partial N}{\partial x} + \frac{\partial N^T}{\partial y}\frac{\partial N}{\partial y} dxdy, \\ [M] &= \sigma \iint_{\Omega^e} N^T N dxdy, \\ \{F\} &= J \iint_\Omega N^T dxdy. \end{aligned} \tag{5}$$

B. Multi-domain design sensitivity equation

In the topology optimization, the optimal material distribution over the design domain is decided by the simple polynomial interpolation function of degree p with respect to the density design variable. In order to define three different domains of the time harmonic excited coil, the iron, and the air in the design domain, three functions are defined as the following equations by two density design variables of ρ_1 and ρ_2:

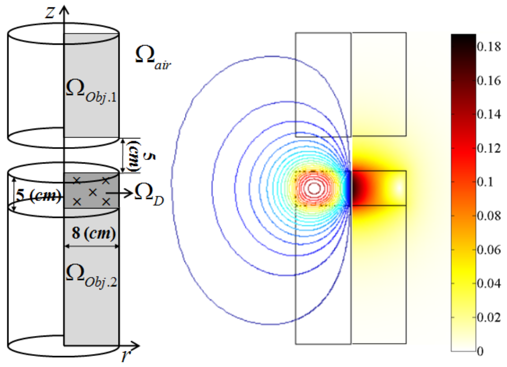
$$\begin{aligned} \mu_r &= \rho_1^p(\rho_2^p(\mu_{coil} - \mu_{iron}) + \mu_{iron} - 1) + 1), \\ J &= J_0 \rho_1^p \rho_2^p, \\ \sigma &= \sigma_0 \rho_1^p \rho_2^p, \end{aligned} \tag{6}$$

where μ_r , μ_{coil} , μ_{iron} , J_0 , and σ_0 are the relative permeability, the relative permeability of coil, the relative permeability of the iron, the initial current density of the coil, and the initial electrical conductivity.

For the purpose of calculating design sensitivity, the performance of objective function is considered as

$$\Psi = \Psi\left(A^*, A^*(\rho_i)\right), \tag{7}$$

where i is the index of ρ_1 and ρ_2.



(a) (b)

Fig. 1. PMF generator of the solenoid type. (a) Initial design; (b) magnetic flux line and magnetic flux density (Tesla).

The derivatives of Eq. (7) with respect to the design variables, $\partial\Psi / \partial\rho_i$, can be calculated by the adjoint variable method (AVM) without a heavy computation problem from finite difference method (FDM) since a great number of design variables are in the design domain:

$$\frac{d\Psi}{d\rho_i} = \frac{\partial\Psi}{\partial\rho_i} + \lambda^* \left(\frac{\partial(F_J - F_{eddy})}{\partial\rho_i} - \frac{\partial K}{\partial\rho_i} A^* \right), \quad (8)$$

where λ^* , F_J and F_{eddy} are the vector of the adjoint variable, and the load vectors of applied source current and induced eddy current, respectively. λ^* can be calculated by the adjoint equation of $K\lambda^* = \left(\partial\Psi / \partial A^*\right)$.

III. Topology Optimization of PMF Generator

A. Optimization problem

Fig. 1. (a) shows the simple solenoid typed PMF generator in the middle of three cylinders. The coil is wound around the *z*-axis, and the current flows into the page as the notation of $\times$. The excitation frequency is 1 (*KHz*).

In this paper, the objective is to focus the PMF on the area of $\Omega_{Obj.1}$ by means of the multi-domain topology optimization in the design domain Ω_D in Fig. 1. Therefore, the objective function is defined as maximizing the magnetic field on $\Omega_{Obj.1}$ and minimizing the fringing field on $\Omega_{Obj.2}$ behind the exciter. The constraint is not considered here. The topology optimization problem takes the form

$$\text{Maximize} \quad \Psi = Energy_{mag.}\Big|_{\Omega_{Obj.1}} - Energy_{mag.}\Big|_{\Omega_{Obj.2}} . \quad (9)$$

The initial design domain is set to be the excited coil since the values of ρ_1 and ρ_2 are initially defined as 1 in Eq. (6). Then the optimization is performed to fine the optimal position of the coil, the iron, and the air.

Fig. 2 shows the optimal material distributions of the excited coil, the iron, and the air. In Fig. 3, almost all of the magnetic flux flows toward $\Omega_{Obj.1}$, and the fringing field on $\Omega_{Obj.2}$ are much less than the initial design.

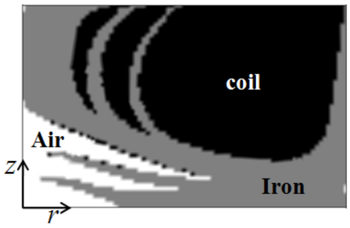


Fig. 2. Optimal design of the excited coil, the iron, and the air.

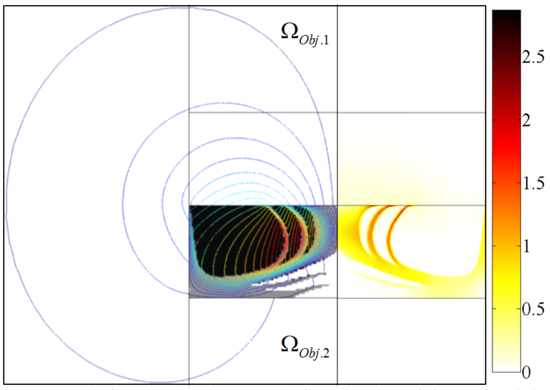


Fig. 3. Magnetic flux line and magnetic flux density of the optimal design.

TABLE I
COMPARISON BETWEEN INITIAL AND OPTIMAL DESIGN

	Initial Design	Optimal Design
Volume of Coil [%]	100	62.96
Volume of Iron [%]	0	26.26
$Energy_{mag.}\big\|_{\Omega_{Obj.1}}$ [%]	100	251.44
$Energy_{mag.}\big\|_{\Omega_{Obj.2}}$ [%]	100	4.92

The optimization results are summarized in Table I. The magnetic energy on $\Omega_{Obj.1}$ is greatly increased, and the leakage on $\Omega_{Obj.2}$ is dramatically reduced.

IV. Conclusion

In this paper, the multi-domain topology optimization in the harmonic magnetic field is performed for the purpose of maximizing PMF on the objective area with less leakage.

V. References

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